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Free transverse vibrations of an elastically connected rectangular plate–membrane complex system

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Abstract

This paper theoretically analyzes undamped free transverse vibrations of an elastically connected rectangular plate–membrane system. Solutions of the problem are formulated by using the Navier method. Natural frequencies of the system in the form of two infinite sequences are determined. Normal mode shapes of vibration expressing two kinds of vibration, synchronous and asynchronous, are presented. The initial-value problem is also solved. In a numerical example, the effect of membrane tension on the natural frequencies of this mixed system is discussed.

1. Introduction

Most real mechanical structures widely used in aeronautical, civil, naval, and mechanical engineering are modelled by simple or complex two-dimensional continuous systems. Fundamental vibration theory of simple two-dimensional continuous systems as membranes and plates is developed in a number of monographs by, for example, Ziemba [1], Solecki and Szymkiewicz [2], Kaliski [3], Leissa [4], Nowacki [5], Timoshenko et al. [6], Osiński [7], Craig [8], and Rao [9], and others. In classical vibration plate theory, two basic analytical methods are applied for analyzing free vibrations of a single rectangular plate, which, as is well known, are the Lévy and Navier methods [1–24]. The Navier method, equivalent to the modal expansion method, is also used for solving free vibration problems of a simply supported rectangular double-plate system [11,25–27]. Double-plate and double-membrane systems are examples of complex two-dimensional continuous systems. The simplest physical models of these structures consist of two parallel plates or membranes, which are connected by an elastic layer of a Winkler type. Free vibrations of the systems under discussion are a subject of scientific interest to numerous investigators [25–37].

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In the present paper, a new model of a complex two-dimensional continuous system is proposed. This is an elastically connected rectangular plate–membrane system [38,39]. Undamped free transverse vibrations of this mixed system are studied by using the classical Navier method. Theoretical vibration analysis of the system is necessary to be performed considering the possibility of application of a membrane as a continuous dynamic vibration absorber (CDVA) in relation to a plate. The system considered has an interesting feature, which enables to change each natural frequency within a certain limited interval as a function only of the membrane tension. At the same time, the other constructional and physical parameters of the system need not be changed. With proper control of the membrane tension, it is possible to avoid a resonance phenomenon or to generate a dynamic vibration absorption phenomenon for the system subjected to harmonic loadings. This can be significant in practical applications. A future publication concerning an elastically connected plate–membrane system will contain the forced vibration analysis showing how to utilize dynamic vibration absorption for suppressing a plate forced vibration.

In another paper, the [40] considers free vibrations of a similar mixed system composed of two one-dimensional continuous models of solids, in an elastically connected beam–string system.

2. Formulation of the problem

The investigated vibratory system model shown in Fig. 1 constitutes a complex continuous system modelled as a rectangular three-layered structure which is composed of isotropic plate, and parallel membrane stretched uniformly by constant tensions applied at the edges, separated by homogeneous massless elastic layer of a Winkler type. It is assumed that both plate and membrane are thin, homogeneous, uniform, and perfectly elastic. For the sake of simplicity of vibration analysis it is also assumed that the plate as well as the membrane are governed by simply supported boundary conditions. In the general case, the system is subjected to arbitrarily distributed transverse continuous loads. Small vibrations of the system with no damping are analyzed.

According to the Kirchhoff–Love plate theory, transverse vibrations of an elastically connected rectangular plate–membrane system are described by the following differential equations:

$$m_1\,\ddot{w}_1 + D_1\,\Delta^2 w_1 + k\,(w_1 - w_2) = f_1, \quad m_2\,\ddot{w}_2 - N_2\,\Delta w_2 + k(w_2 - w_1) = f_2, \tag{1}$$

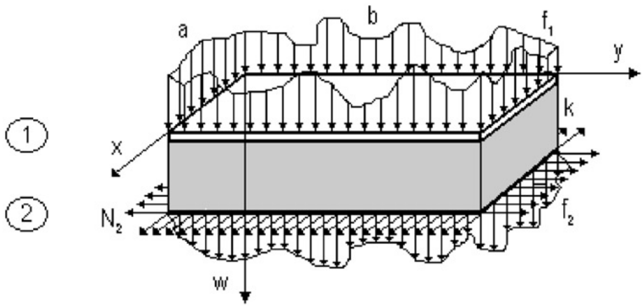

Fig. 1. The physical model of an elastically connected rectangular plate–membrane complex system.

where $w_i = w_i(x, y, t)$ is the transverse plate (membrane) displacement; $f_i = f_i(x, y, t)$ is the exciting distributed load; x, y, t are the space co-ordinates and the time; D_1 is the flexural rigidity of the plate; E_1 is Young's modulus of elasticity for the plate; N_2 is the uniform constant tension per unit length for the membrane; k is the stiffness modulus of a Winkler elastic layer; a, b, h_i are the plate (membrane) dimensions; v_1 is the Poisson's ratio; ρ_i is the mass density;

$$D_1 = E_1 h_1^3[12(1 - v_1^2)]^{-1}, \quad m_i = \rho_i h_i, \quad \dot{w}_i = \partial w_i/\partial t, \quad i = 1, 2,$$

$$\Delta^2 w_1 = \partial^4 w_1/\partial x^4 + 2\partial^4 w_1/\partial x^2 \partial y^2 + \partial^4 w_1/\partial y^4, \quad \Delta w_2 = \partial^2 w_2/\partial x^2 + \partial^2 w_2/\partial y^2.$$

The subscripts 1 and 2 refer to the plate, denoted by the index 1, and the membrane, denoted by the index 2, respectively.

The boundary conditions for the simply supported plate and membrane are as follows:

$$\begin{aligned} &w_1(0, y, t) = w_1(a, y, t) = w_1(x, 0, t) = w_1(x, b, t) = 0, \\ &\partial^2 w_1/\partial x^2\big|_{(0, y, t)} = \partial^2 w_1/\partial x^2\big|_{(a, y, t)} = \partial^2 w_1/\partial y^2\big|_{(x, 0, t)} = \partial^2 w_1/\partial y^2\big|_{(x, b, t)} = 0, \\ &w_2(0, y, t) = w_2(a, y, t) = w_2(x, 0, t) = w_2(x, b, t) = 0. \end{aligned} \tag{2}$$

The initial conditions may be written in the following general form:

$$w_i(x, y, 0) = w_{i0}(x, y), \quad \dot{w}_i(x, y, 0) = v_{i0}(x, y), \quad i = 1, 2. \tag{3}$$

3. Solution of the free vibration problem

Free vibrations of a rectangular plate-membrane system (see Fig. 2) are governed by the following homogeneous partial differential equations [38,39]:

$$m_1\ddot{w}_1 + D_1\Delta^2 w_1 + k(w_1 - w_2) = 0, \quad m_2\ddot{w}_2 - N_2\Delta w_2 + k(w_2 - w_1) = 0. \tag{4}$$

The above equation system with the boundary conditions (2) can be solved by the Navier method equivalent to the modal expansion method assuming solutions in the form

$$\begin{aligned} w_1(x, y, t) &= \sum_{m,n=1}^{\infty} W_{mn}(x, y)\, S_{1mn}(t) = \sum_{m,n=1}^{\infty} \sin(a_m x)\sin(b_n y)\, S_{1mn}(t), \\ w_2(x, y, t) &= \sum_{m,n=1}^{\infty} W_{mn}(x, y)\, S_{2mn}(t) = \sum_{m,n=1}^{\infty} \sin(a_m x)\sin(b_n y)\, S_{2mn}(t), \end{aligned} \tag{5}$$

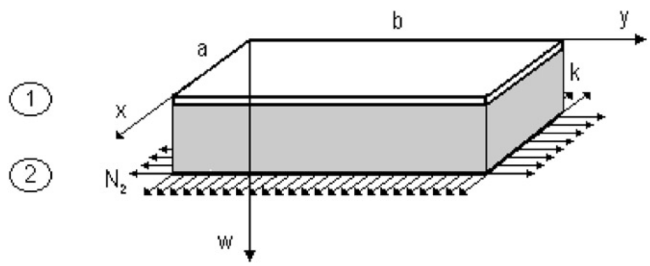

Fig. 2. The physical model of an elastically connected rectangular plate–membrane system analyzed for free vibrations.

where $S_{imn}(t)$, $(i = 1, 2)$ are the unknown time functions;

$$W_{mn}(x, y) = X_m(x)\, Y_n(y) = \sin(a_m x)\sin(b_n y),$$
$$X_m(x) = \sin(a_m x), \quad Y_n(y) = \sin(b_n y), \quad m, n = 1, 2, 3, \ldots,$$
$$a_m = a^{-1}m\pi, \quad b_n = b^{-1}n\pi, \quad k^2_{mn} = a^2_m + b^2_n = \pi^2[(a^{-1}m)^2 + (b^{-1}n)^2]. \tag{6}$$

$W_{mn}(x, y)$ are the known mode shape functions satisfying the corresponding boundary conditions (2) for the simply supported plate and membrane as well as the homogeneous differential equations (4).

Substituting solutions (5) into Eqs. (4) gives the following expressions:

$$\sum_{m,\,n=1}^{\infty} [\ddot{S}_{1mn} + (D_1 k^4_{mn} + k)m_1^{-1}S_{1mn} - km_1^{-1}S_{2mn}]W_{mn} = 0,$$
$$\sum_{m,\,n=1}^{\infty} \left[\ddot{S}_{2mn} + (N_2 k^2_{mn} + k)m_2^{-1}S_{2mn} - km_2^{-1}S_{1mn}\right]W_{mn} = 0,$$

from which a set of ordinary differential equations for the unknown time functions is obtained

$$\ddot{S}_{1mn} + \Omega^2_{11mn}S_{1mn} - \Omega^2_{10}S_{2mn} = 0, \quad \ddot{S}_{2mn} + \Omega^2_{22mn}S_{2mn} - \Omega^2_{20}S_{1mn} = 0, \tag{7}$$

where

$$\Omega^2_{11mn} = (D_1 k^4_{mn} + k)m_1^{-1}, \quad \Omega^2_{22mn} = (N_2 k^2_{mn} + k)m_2^{-1},$$
$$\Omega^2_{i0} = km_i^{-1}, \quad \Omega^4_{120} = \Omega^2_{10}\Omega^2_{20} = k^2(m_1 m_2)^{-1}, \quad i = 1, 2.$$

Ω_{iimn} $(i = 1, 2)$ and Ω_{120} denote the partial and coupling frequency of the system, respectively. The solutions of Eqs. (7) are as follows:

$$S_{1mn}(t) = C_{mn}e^{i\omega_{mn}t}, \quad S_{2mn}(t) = D_{mn}e^{i\omega_{mn}t}, \quad i = (-1)^{1/2}, \tag{8}$$

where ω_{mn} is the natural frequency of the system. Introducing them into Eqs. (7) results in the system of algebraic equations for unknown constants C_{mn}, D_{mn}:

$$(\Omega^2_{11mn} - \omega^2_{mn})C_{mn} - \Omega^2_{10}D_{mn} = 0, \quad (\Omega^2_{22mn} - \omega^2_{mn})D_{mn} - \Omega^2_{20}C_{mn} = 0. \tag{9}$$

For non-trivial solutions of the above equations, the determinant of the system coefficient matrix is set equal to zero, yielding the following frequency equation:

$$\omega^4_{mn} - (\Omega^2_{11mn} + \Omega^2_{22mn})\omega^2_{mn} + (\Omega^2_{11mn}\Omega^2_{22mn} - \Omega^4_{120}) = 0 \tag{10}$$

or

$$\begin{aligned}\omega^4_{mn} &- [(D_1 k^4_{mn} + k)m_1^{-1} + (N_2 k^2_{mn} + k)m_2^{-1}]\omega^2_{mn} \\ &+ k^2_{mn}[D_1 N_2 k^4_{mn} + k(D_1 k^2_{mn} + N_2)](m_1 m_2)^{-1} = 0.\end{aligned} \tag{11}$$

Since the discriminant of this biquadratic algebraic equation is positive

$$D = (\Omega^2_{11mn} + \Omega^2_{22mn})^2 - 4(\Omega^2_{11mn}\Omega^2_{22mn} - \Omega^4_{120}) = (\Omega^2_{11mn} - \Omega^2_{22mn})^2 + 4\Omega^4_{120} > 0$$

and the relationships mentioned below are also satisfied:

$$(\Omega^2_{11mn}\Omega^2_{22mn} - \Omega^4_{120}) > 0, \quad (\Omega^2_{11mn} + \Omega^2_{22mn}) > D^{1/2},$$

and thus the frequency equation (10) has two different, real, positive roots $\omega^2_{1,\,2mn}$:

$$\omega^2_{1,\,2mn} = 0.5\left\{(\Omega^2_{11mn} + \Omega^2_{22mn}) \mp [(\Omega^2_{11mn} - \Omega^2_{22mn})^2 + 4\Omega^4_{120}]^{1/2}\right\}, \quad \omega_{1mn} < \omega_{2mn}. \tag{12}$$

Two infinite sequences of the natural frequencies ω_{1mn}, ω_{2mn} are obtained in the form

$$\omega^2_{1,\,2mn} = 0.5\{[(D_1k^4_{mn} + k)m_1^{-1} + (N_2k^2_{mn} + k)m_2^{-1}] \mp ([(D_1k^4_{mn} + k)m_1^{-1} + (N_2k^2_{mn} + k)m_2^{-1}]^2 - 4k^2_{mn}(m_1m_2)^{-1}[D_1N_2k^4_{mn} + k(D_1k^2_{mn} + N_2)])^{1/2}\}, \tag{13}$$

The time functions (8) may be written as follows:

$$S_{1mn}(t) = C_{1mn}e^{i\omega_{1mn}t} + C_{2mn}e^{-i\omega_{1mn}t} + C_{3mn}e^{i\omega_{2mn}t} + C_{4mn}e^{-i\omega_{2mn}t},$$

$$S_{2mn}(t) = D_{1mn}e^{i\omega_{1mn}t} + D_{2mn}e^{-i\omega_{1mn}t} + D_{3mn}e^{i\omega_{2mn}t} + D_{4mn}e^{-i\omega_{2mn}t},$$

or in more useful alternative trigonometric form

$$S_{1mn}(t) = \sum_{i=1}^{2} T_{imn}(t) = \sum_{i=1}^{2}[A_{imn}\sin(\omega_{imn}t) + B_{imn}\cos(\omega_{imn}t)],$$

$$S_{2mn}(t) = \sum_{i=1}^{2} a_{imn}T_{imn}(t) = \sum_{i=1}^{2}[A_{imn}\sin(\omega_{imn}t) + B_{imn}\cos(\omega_{imn}t)]a_{imn}, \tag{14}$$

where

$$T_{imn}(t) = A_{imn}\sin(\omega_{imn}t) + B_{imn}\cos(\omega_{imn}t), \quad m, n = 1, 2, 3, \ldots, \tag{15}$$

$$a_{imn} = (D_1k^4_{mn} + k - m_1\omega^2_{imn})k^{-1} = k(N_2k^2_{mn} + k - m_2\omega^2_{imn})^{-1}$$
$$= \Omega_{10}^{-2}(\Omega^2_{11mn} - \omega^2_{imn})$$
$$= \Omega^2_{20}(\Omega^2_{22mn} - \omega^2_{imn})^{-1},$$
$$k^2_{mn} = \pi^2[(a^{-1}m)^2 + (b^{-1}n)^2], \quad i = 1, 2. \tag{16}$$

It is easy to show that the coefficients a_{imn} may be presented in the form

$$a_{1,\,2mn} = 0.5\Omega_{10}^{-2}\{(\Omega^2_{11mn} - \Omega^2_{22mn}) \pm [(\Omega^2_{11mn} - \Omega^2_{22mn})^2 + 4\Omega^4_{120}]^{1/2}\}, \quad a_{1mn} > 0, \quad a_{2mn} < 0,$$

$$a_{1mn}a_{2mn} = -m_1m_2^{-1} = -M_1M_2^{-1} = -\Omega_{10}^{-2}\Omega^2_{20}, \quad M_i = abm_i = abh_i\rho_i.$$

It is seen that the coefficient a_{1mn}, dependent on lower natural frequency ω_{1mn}, is always positive while a_{2mn}, dependent on higher frequency ω_{2mn}, is always negative.

Finally, the free transverse vibrations of an elastically connected rectangular plate–membrane system may be written in the following form:

$$\begin{aligned} w_1(x,\, y,\, t) &= \sum_{m,\, n=1}^{\infty} W_{mn}(x,\, y) \sum_{i=1}^{2} T_{imn}(t) = \sum_{m,\, n=1}^{\infty} \sum_{i=1}^{2} W_{1imn}(x,\, y) T_{imn}(t) \\ &= \sum_{m,\, n=1}^{\infty} \sin(a_m x)\sin(b_n y) \sum_{i=1}^{2} [A_{imn}\sin(\omega_{imn} t) + B_{imn}\cos(\omega_{imn} t)], \\ w_2(x,\, y,\, t) &= \sum_{m,\, n=1}^{\infty} W_{mn}(x,\, y) \sum_{i=1}^{2} a_{imn} T_{imn}(t) = \sum_{m,\, n=1}^{\infty} \sum_{i=1}^{2} W_{2imn}(x,\, y) T_{imn}(t) \\ &= \sum_{m,\, n=1}^{\infty} \sin(a_m x)\sin(b_n y) \sum_{i=1}^{2} [A_{imn}\sin(\omega_{imn} t) + B_{imn}\cos(\omega_{imn} t)] a_{imn}, \end{aligned} \tag{17}$$

where

$$\begin{aligned} W_{1imn}(x,\, y) &= W_{mn}(x,\, y) = \sin(a_m x)\sin(b_n y), \\ W_{2imn}(x,\, y) &= a_{imn} W_{mn}(x,\, y) = a_{imn}\sin(a_m x)\sin(b_n y). \end{aligned} \tag{18}$$

The functions $W_{1imn}(x,\, y)$, $W_{2imn}(x,\, y)$ are the natural mode shapes of vibration of the plate–membrane system corresponding to two infinite sequences of the natural frequencies ω_{imn}. General mode shapes for the first four pairs of the natural frequencies are presented in Fig. 3. It is seen that an elastically connected plate-membrane system executes two types of vibrating motion: synchronous vibrations ($i = 1$; $a_{1mn} > 0$) with lower frequencies ω_{1mn} and asynchronous vibrations ($i = 2$; $a_{2mn} < 0$) with higher frequencies ω_{2mn}. The mode shapes obtained for a system considered are the same as those determined for a simply supported double-plate system [11,27], and for a double-membrane system [11,37]. It should also be noted that the nature of free vibrations is identical for all these three systems as a consequence of defining the same boundary conditions.

The unknown constants A_{imn}, B_{imn} in expressions (17) are calculated by solving the initial-value problem. To make it possible, knowledge of the orthogonality condition of mode shape functions is necessary. In this case the orthogonality condition has the classical form [2,3,11]

$$\begin{aligned} \int_0^a \int_0^b W_{kl} W_{mn}\, \mathrm{d}x\, \mathrm{d}y &= \int_0^a \sin(a_k x)\sin(a_m x)\, \mathrm{d}x \int_0^b \sin(b_l y)\sin(b_n y)\, \mathrm{d}y = c\delta_{klmn}, \\ c = c_{mn}^2 &= \int_0^a \int_0^b W_{mn}^2\, \mathrm{d}x\, \mathrm{d}y = \int_0^a \sin^2(a_m x)\, \mathrm{d}x \int_0^b \sin^2(b_n y)\, \mathrm{d}y = 0.25ab, \end{aligned} \tag{19}$$

where δ_{klmn} is the Kronecker delta function: $\delta_{klmn} = 0$ for $k \neq m$ or $l \neq n$, and $\delta_{klmn} = 1$ for $k = m$ and $l = n$,

Substituting solutions (17) into the initial conditions (3), and then performing the known usual transformation procedure and applying the orthogonality condition (19), the following formulae

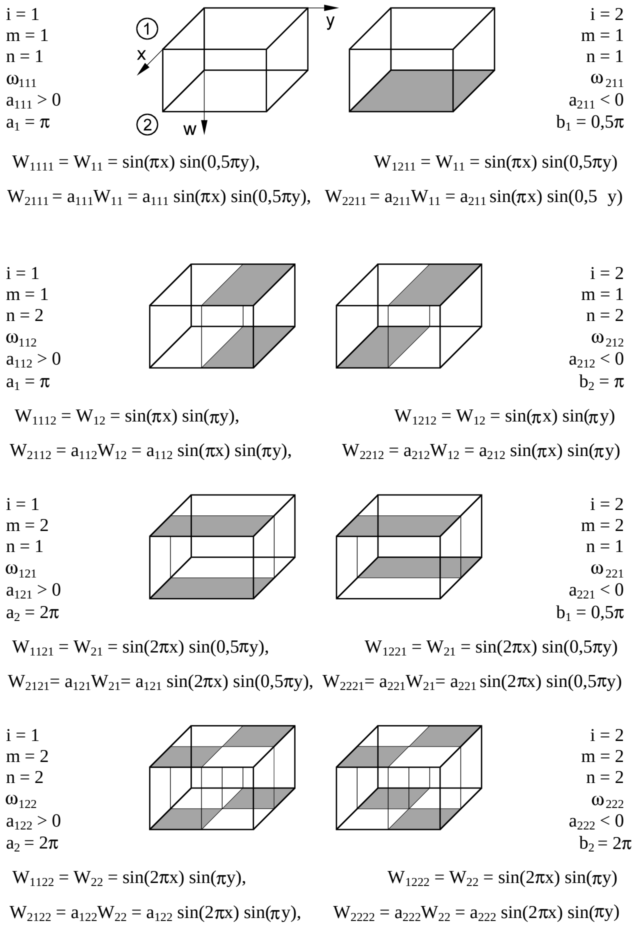

Fig. 3. The general mode shapes of vibration of an elastically connected rectangular plate–membrane system corresponding to the first four pairs of the natural frequencies ω_{imn} ($i=1, 2$; $m,n=1, 2$). The mode shapes for $i=1$ and $i=2$ express the synchronous ($a_{1mn} > 0$, ω_{1mn}) and asynchronous ($a_{2mn} < 0$, ω_{2mn}) free vibrations, respectively.

for evaluating A_{imn}, B_{imn} are obtained:

$$
\begin{aligned}
A_{1mn} &= (\omega_{1mn} z_{1mn})^{-1} \int_0^a \int_0^b (a_{2mn} v_{10} - v_{20}) \sin(a_m x) \sin(b_n y)\, \mathrm{d}x\, \mathrm{d}y, \\
A_{2mn} &= (\omega_{2mn} z_{2mn})^{-1} \int_0^a \int_0^b (a_{1mn} v_{10} - v_{20}) \sin(a_m x) \sin(b_n y)\, \mathrm{d}x\, \mathrm{d}y, \\
B_{1mn} &= z_{1mn}^{-1} \int_0^a \int_0^b (a_{2mn} w_{10} - w_{20}) \sin(a_m x) \sin(b_n y)\, \mathrm{d}x\, \mathrm{d}y, \\
B_{2mn} &= z_{2mn}^{-1} \int_0^a \int_0^b (a_{1mn} w_{10} - w_{20}) \sin(a_m x) \sin(b_n y)\, \mathrm{d}x\, \mathrm{d}y, \qquad (20)
\end{aligned}
$$

where

$$z_{2mn} = -z_{1mn} = (a_{1mn} - a_{2mn})c = 0.25ab(a_{1mn} - a_{2mn}) = 0.25ab\Omega_{10}^{-2}(\omega_{2mn}^2 - \omega_{1mn}^2).$$

It can be shown that the free vibration analysis made here for a rectangular plate–membrane system is analogous to that for a simply supported double-plate [27], and a double-membrane system [37].

4. Numerical example

The purpose of this simple example is to demonstrate the effect of a membrane tension N_2 on the natural frequencies of the system.

The following values of the parameters characterizing properties of the system are used in the numerical calculations:

$$a = 1\,\text{m}, \quad b = 2\,\text{m}, \quad E_1 = 1\times 10^8\,\text{N}\,\text{m}^{-2}, \quad h_1 = 1\times 10^{-2}\,\text{m}, \quad h_2 = 4\times 10^{-3}\,\text{m},$$
$$k = 1\times 10^4\,\text{N}\,\text{m}^{-3}, \quad m_1 = \rho_1 h_1 = 50\,\text{kg}\,\text{m}^{-2}, \quad m_2 = \rho_2 h_2 = 1\,\text{kg}\,\text{m}^{-2}, \quad \nu_1 = 0.3,$$
$$N_2 = 0,\ 100,\ 200,\ 300,\ 400,\ 500\,\text{N}\,\text{m}^{-1}, \quad \rho_1 = 5\times 10^3\,\text{kg}\,\text{m}^{-3}, \quad \rho_2 = 2.5\times 10^2\,\text{kg}\,\text{m}^{-3}.$$

The free vibrations of the system discussed are described by relations (17):

$$w_1(x, y, t) = \sum_{m,\,n=1}^{\infty}\sum_{i=1}^{2} W_{1imn}(x, y)T_{imn}(t)$$
$$= \sum_{m,\,n=1}^{\infty}\sin(a_m x)\sin(b_n y)\sum_{i=1}^{2}[A_{imn}\sin(\omega_{imn}t) + B_{imn}\cos(\omega_{imn}t)],$$
$$w_2(x, y, t) = \sum_{m,\,n=1}^{\infty}\sum_{i=1}^{2} W_{2imn}(x, y)T_{imn}(t)$$
$$= \sum_{m,\,n=1}^{\infty}\sin(a_m x)\sin(b_n y)\sum_{i=1}^{2}[A_{imn}\sin(\omega_{imn}t) + B_{imn}\cos(\omega_{imn}t)]a_{imn}.$$

The general natural mode shapes of vibration $W_{1imn}(x)$ and $W_{2imn}(x)$ are (18)

$$W_{1imn}(x, y) = \sin(a_m x)\sin(b_n y), \quad W_{2imn}(x, y) = a_{imn}\sin(a_m x)\sin(b_n y),$$

where

$$a_m = a^{-1}m\pi, \quad b_n = b^{-1}n\pi, \quad a_{1mn} > 0, \quad a_{2mn} < 0.$$

Exemplar mode shapes corresponding to the first four pairs of the natural frequencies ω_{imn} are shown in Fig. 3. The mode shapes for $i=1$ and 2 express the synchronous ($a_{1mn} > 0$, ω_{1mn}) and asynchronous ($a_{2mn} < 0$, ω_{2mn}) free vibrations of the system, respectively.

The natural frequencies ω_{imn} are evaluated from relations (12) and (13) as functions of a tension magnitude N_2. Results of the calculations for i=1, 2 and m,n=1, 2 are presented in Table 1 and in Fig. 4. An evident influence of membrane tension on the frequencies of the system is observed. In any case, increasing N_2 causes an increasing of ω_{imn}. However this influence of the membrane

Table 1
Natural frequencies of rectangular plate–membrane system ω_{imn} (s^{-1})

N_2	0	100	200	300	400	500
ω_{111}	5.2	7.0	8.2	9.0	9.7	10.2
ω_{211}	101.0	106.8	112.4	117.7	122.8	127.6
ω_{112}	8.4	10.1	11.3	12.0	12.6	13.0
ω_{212}	101.0	110.2	118.7	126.7	134.2	141.3
ω_{121}	11.0	19.4	20.3	20.8	21.1	21.4
ω_{221}	101.0	119.8	136.0	150.6	163.9	175.4
ω_{122}	15.5	22.5	23.3	23.7	24.0	24.2
ω_{222}	101.0	122.8	141.3	157.8	172.7	186.4

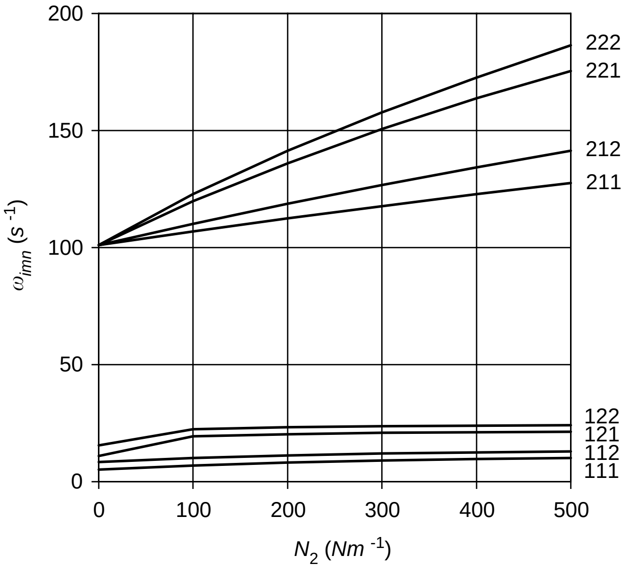

Fig. 4. The natural frequencies of plate–membrane system ω_{imn} ($i=1,\ 2$; $m,n=1,\ 2$) as a function of membrane tension N_2.

tension on the particular frequencies is different, and the effect of N_2 on the asynchronous frequencies ω_{2mn} is greater than on the synchronous ones ω_{1mn}.

It can be seen that the mixed system discussed has an interesting feature, which allows each natural frequency to change as a function of membrane tension, whilst the other constructional and physical parameters of the system can remain unchanged. Selecting suitable tension of membrane gives desirable values of the system frequencies in certain limited domains, so that it is possible, for instance, to avoid resonance phenomena or to generate a dynamic vibration absorption phenomenon. As is well known, vibration absorption can be used to suppress excessive forced vibration amplitudes [11,37]. This fact can have significance in practical applications of such mixed complex systems.

5. Conclusions

In this study, free transverse vibrations of an elastically connected rectangular plate–membrane system are analyzed theoretically. The vibratory system model considered comprises a three-layered structure which is composed of a thin plate, a massless elastic layer modelled as a homogeneous Winkler-type foundation, and a parallel membrane stretched uniformly by suitable constant tensions applied at the edges. The problem is solved by using the classical Navier method equivalent to the modal expansion method. Two infinite sequences of the natural frequencies and corresponding mode shape functions expressing synchronous and asynchronous vibrations of the system are obtained. It should be noted that the natural frequencies of the system may be varied with a change of membrane tensions without the necessity to vary parameters characterizing physical and geometrical properties of the system. This possibility is of great practical importance. The final form of free vibrations is found by solving the generally formulated initial-value problem. Solutions for the system discussed are analogous with those obtained for an elastically connected rectangular simply supported double-plate system [11,27], and for a similar system of two membranes [11,37].

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